2. Define P（n）: “(q, w) \* (q, )”, where |w| = n

Base Case： n = 0

w =

(q, w) (q, ) \* (q, )

Inductive Step：

Let n = k, Assume P(k), i.e., (q, w) \* (q, ), where |w| = k

Let w’ = wa. We want to show P(k+1), i.e., (q, w’) \* (q, )

(q, w’) (q, ) (q, a) (By inductive hypothesis)

Then, (q, a)(q, )

So P(k+1) holds.

3. The automation accepts the language that ends with “01” or “00”.

4.

a)

(0+1)\* 1 (0+1) (0+1) (0+1) (0+1) (0+1) (0+1)

b)

(0+10)\* (11+) (0+10)\*

c)

(0+10)\* (1)\*

5.

a)

The statement is False. Disprove by giving counterexample.

Let , R = a, S = b

Proof for ab L((a+b)\*):

a L((a+b)), b L((a+b)), so abL((a+b)\*)

Proof for ab L(a\* + b\*):

ab L(a\*) and ab L(b\*), so ab L(a\* + b\*)

Since abL((a+b)\*) but ab L(a\* + b\*), the statement is false.

b)

The statement is False. Disprove by giving counterexample.

Let , R = a, S = b

Firstly, we know that L((aa\*b)\*), because there could be zero replication of aa\*b.

However, L((ab + a)\*ab) because the language must contain at least “ab”. (since (ab + a)\*)

Since (ab + a)\*)), but L((ab + a)\*ab), the statement is false.

c)

The statement is False. Disprove by giving counterexample.

Let , R = a, S = b

Firstly, we know that a(a), then a (a + b). Therefore, a (a + b)\*).

Secondly, a L((a\*b)), then a L((a\*b)\*).

All string in the language, L((a\*b)\*) must contain either at least one b or is .

Since , but L((a\*b)\*), the statement is false.

d)

The statement is False. Disprove by giving counterexample.

Let , R = a, S = b

Firstly, we know that (ab + b), then ba (b(ab + b)\*a).

Secondly, we know that ((aa\*b)\*) and ((a)\*), then the only possible string of the language, L(aa\*b(aa\*b)\*), of length two is ab, so ba L(aa\*b(aa\*b)\*).

Since (b(ab + b)\*a), but L(aa\*b(aa\*b)\*), the statement is false.

6.

N(k) =

Explanation: If k is greater than 4, it is clear that the string that is accepted by the automaton must end with 11 or 010 or 001.

Case 1: if the string that is accepted by the automaton ends with 11. It means that we can get the target string of length k by concatenating each accepted string of length (k-2) with 11.

Case 2: if the string that is accepted by the automaton ends with 010 or 001. It means that we can get the target string of length k by concatenating each accepted string of length (k-3) with 010 or 001.

Therefore, if k is greater than 4, N(k) = N(n-2) + 2N(k-3).

Then N(14) = N(12) + 2N(11)

= N(10) + 2N(9) + 2(N(9)+2N(8))

= N(10) + 4N(9) + 4N(8)

= N(8) + 2N(7) + 4(N(7)+2N(6)) + 4(N(6)+2N(5))

…

= 198

7.

a)

1. Suppose L is regular.

2. Let n be the pumping constant.

3. Choose w = 0n1m0n (Note that w ∈ L and m, n )

4. By PL, w can be factored into xyz such that |xy| ≤ n, |y| > 0 and, for all i ≥ 0, xyiz ∈ L 5. Since |xy| ≤ n and |y| > 0, x = 0p, y = 0q, z = 1m0n where p + q = n, p ≥ 0, q > 0, then p < n.

6. Consider i = 0, then xyiz = xz = 0p1m0n ∈ L. We have obtained a contradiction of the assumed regularity of L. (Since p < n )

7. Hence, L is not regular

b)

1. Suppose L is regular.

2. Let n be the pumping constant.

3. Since L = {w∈{0,1}\*|w is palindrome}, we choose w = 0n1m0n (Note that w ∈ L and m, n )

4. By PL, w can be factored into xyz such that |xy| ≤ n, |y| > 0 and, for all i ≥ 0, xyiz ∈ L 5. Since |xy| ≤ n and |y| > 0, x = 0p, y = 0q, z = 1m0n where p + q = n, p ≥ 0, q > 0, then p < n.

6. Consider i = 0, then xyiz = xz = 0p1m0n ∈ L. We have obtained a contradiction of the assumed regularity of L. (Since p < n )

7. Hence, L is not regular